Design and Evaluation of adaptive Filter Using Normalized LMS Algorithm

Abstract

Noise problems in the environment have gained attention due to the tremendous growth of technology that has led to noisy engines, heavy machinery, high speed wind buffeting and other noise sources. The problem of controlling the noise level has become the focus of a tremendous amount of research over the years. In last few years various adaptive algorithms are developed for noise cancellation. In this paper we present an implementation of LMS (Least Mean Square) and NLMS (Normalized Least Mean Square algorithms on MATLAB . We simulate the adaptive filter in MATLAB with a noisy tone signal and white noise signal and analyze the performance of algorithms.

Keywords: Adaptive filter; convergence speed; LMS; Mean Squared Error; NLMS.

Introduction

In the process of transmission of information from the source to receiver side in all the channels, noise from the surroundings automatically gets added to the signal. This acoustic noise [1] picked up by microphone is undesirable, as it reduces the perceived quality or intelligibility of the audio signal. The problem of effective removal or reduction of noise is an active area of research [2]. The usage of adaptive filters is one of the most popular proposed solutions to reduce the signal corruption caused by predictable and unpredictable noise added to the source signal. An adaptive filter [3] has the property of self-modifying its frequency response to change the behavior in time domain, allowing the filter to adapt the response to the input signal characteristics change. Because of this capability, overall performance and the construction flexibility, the adaptive filters have been employed in many different applications, some of the most important are: telephonic echo cancellation [1], radar signal processing, navigation systems, communications channel equalization and biometrics signals processing. The purpose of an adaptive filter in noise cancellation is to remove the noise from a signal adaptively to improve the signal to noise ratio. Figure 1 shows Adaptive Noise Cancellation (ANC) system [4]. The discrete adaptive filter processed the reference signal x(n) to produce the output signal y(n) by a convolution with filter's weights w(n).The filter output y(n) is subtracted from d(n) to obtain an estimation error e(n). The primary sensor receives noise x1(n) which has correlation with noise x(n) in an unknown way. The objective here is to minimize the error signal e(n). This error signal is used to incrementally adjust the filter's weights for the next time instant. The basic adaptive algorithms which widely used for performing weight updation of an adaptive filter are: the LMS (Least Mean Square), NLMS (Normalized Least Mean Square) and the RLS (Recursive Least Square) algorithm [5]. Among all adaptive algorithms LMS has probably become the most popular for its robustness, good tracking capabilities and simplicity in stationary environment.

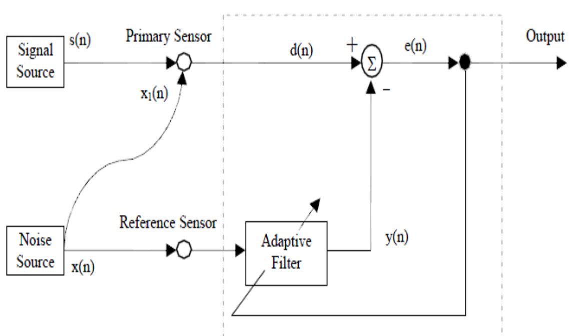

Figure 1. adaptive noise canceller

Design of adaptive filter

Steepest Descent Algorithm

An adaptive filter is required to find a solution for its tap-weight vector that satisfies the normal equation. A procedure is to use the method of steepest descent, which is one of the oldest methods of optimization.

1. Initial values of w(0) are chosen arbitrarily i.e. initial guess as to where the minimum point of the error-performance surface may be located. Typically w(0) = null vector.

2. Using this, we compute the gradient vector, defined as the gradient of mean squared error J(n) wrt w(n) at time n (nth iteration).

3. We compute the next guess at the tap-weight vector by making a change in the initial or present guess in a direction opposite to that of the gradient vector.

4. Go back to step 2 and repeat the process.

$w(n+1) = w(n) + 0.5\mu [-\nabla(n)]$, μ = positive real-valued constant …..(1)

$\nabla(n) = -2p + 2Rw(n)$… .(2)

For the application of the steepest-descent algorithm, we assume that the correlation matrix R and cross-correlation matrix p are known[3]. By taking their values from the given two filter signals named as reference and primary which is mention in the M file.

$w(n+1) = w(n) + \mu[p - Rw(n)]$ n = 0, 1, . (3)

We observe that the parameter μ controls the size of the incremental correction applied to the tap-weight vector as we proceed from one iteration to the next. Therefore, μ is referred to as the step-size parameter or weighting constant[4]. The equation (3) describes the mathematical formulation of the steepest-descent algorithm or also referred to as the deterministic gradient algorithm.

Least-Mean-Squares (LMS) Algorithm

The LMS algorithm remedies this problem by adapting the filter weighs according to the incoming audio data as it is being received. The LMS algorithm is robust enough for a variety of signal conditions due to its adaptive nature. The LMS algorithm involves the computation of the output of a linear filter in response to the noise reference and the generation of the estimation error between this output and the desired respons[6]e. The estimation error is used in the adjustment of the filter weighs. If it were possible to make exact measurements of the gradient vector at each iteration, and if the step-size parameter μ is suitably chosen, then the tap-weight vector computed by using the method of steepest-descent would indeed converge to the optimum solution . A significant feature of LMS is its simplicity; it does not require measurements of the pertinent correlation functions, nor does it require matrix inversion.

gradient vector, $\nabla(n) = -2p + 2Rw(n)$

To estimate this, we estimate the correlation matrix R and cross-correlation matrix p by instantaneous estimates i.e.

$R'(n) = u(n)u(n)$..(4)

$p'(n) = u(n)\, d^*(n)$…(5)

Correspondingly, the instantaneous estimate of the gradient-vector is

$\nabla(n) = -2\, u(n)\, d^*(n) + 2\, u(n)uH(n)w(n)$..(6).

Substituting this estimate in the steepest-descent algorithm, equation (3), we get a new recursive relation for updating the tap-weight vector:

$w'(n+1) = w'(n)+\mu u(n)[d^*(n) - uH(n)w'(n)]$ …(7)

Equivalently the LMS update equation can be written in the form of a pair of relations:

$e(n) = d(n) - uH(n)w'(n)$ …(8)

$w'(n+1) = w'(n)+\mu u(n)e^*(n)$ …(9)

Excess mean squared error is defined as the amount by which the actual value of $J(\infty)$ is greater than Jmin, Misadjustment .The misadjustment M is defined as the dimensionless ratio of the steady-state value of the average excess mean-squared error to the minimum mean squared error. It can be shown that

$M=\mu\sum\lambda i /2-\mu\sum\lambda i$ ….(10)

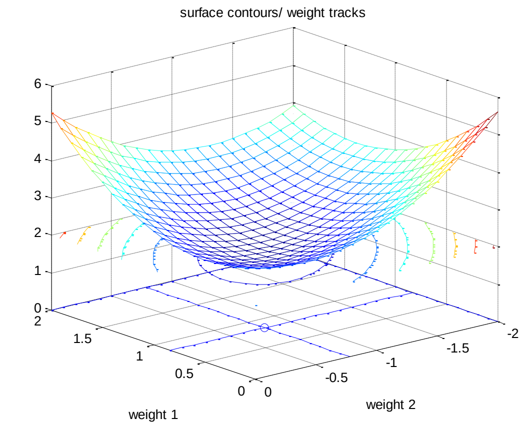

Figure (2) surface contour/ weight tracks

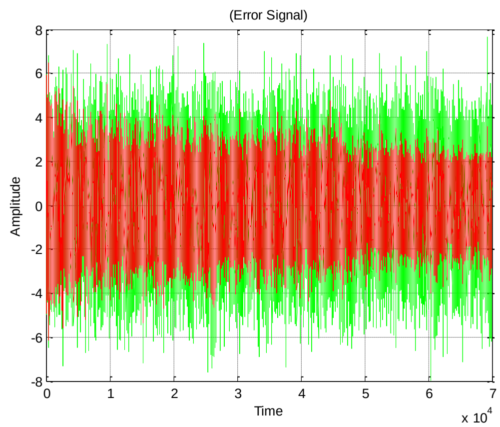

Figure 3 .Error signal

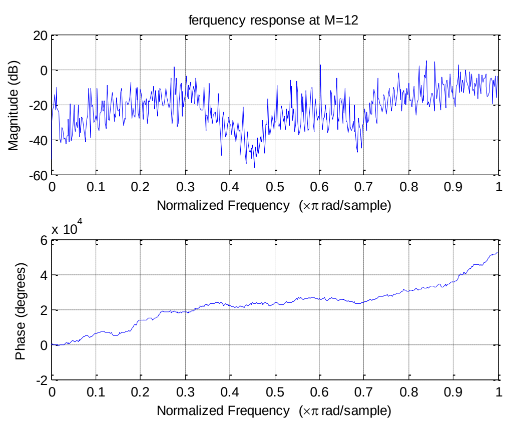

Figure 4. Frequency response 1

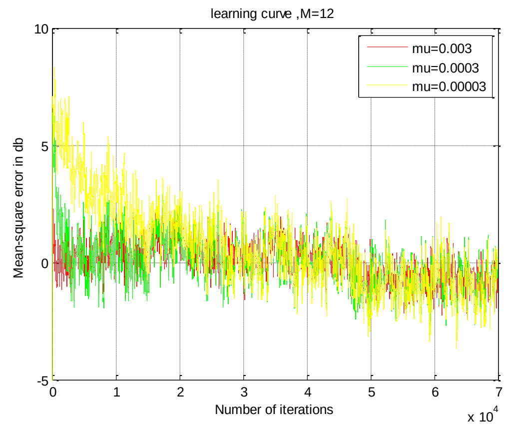

Figure 5. learning curve 1

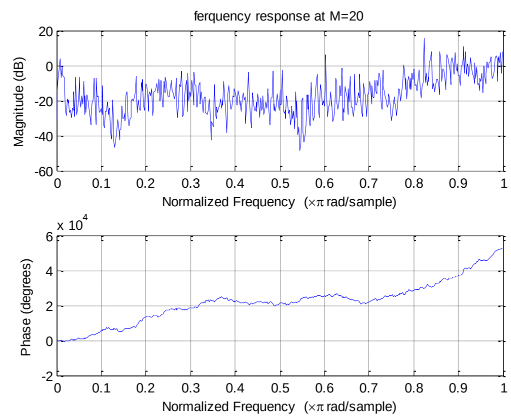

Figure 6. Frequency response 2

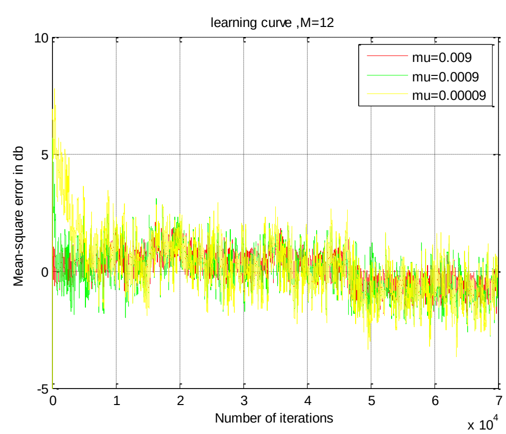

Figure 7. learning curve 2

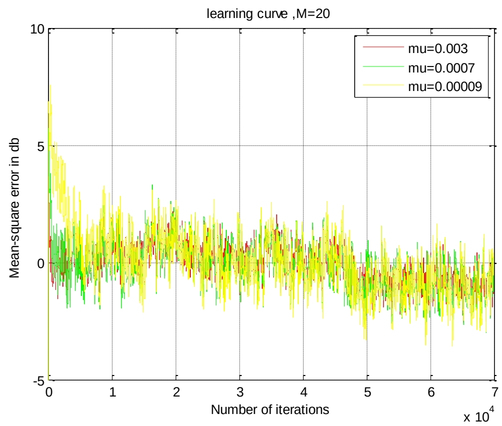

Figure 8. learning curve 3

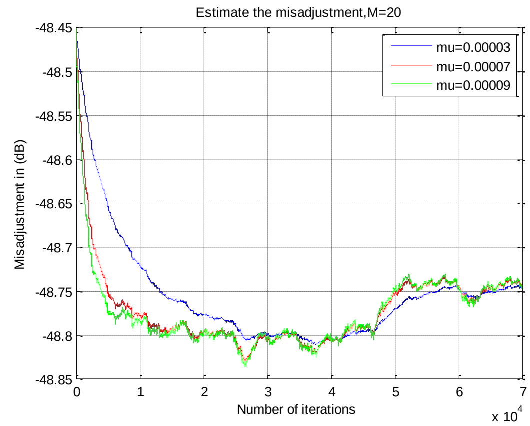

Figure 11. misadjustment curve 2

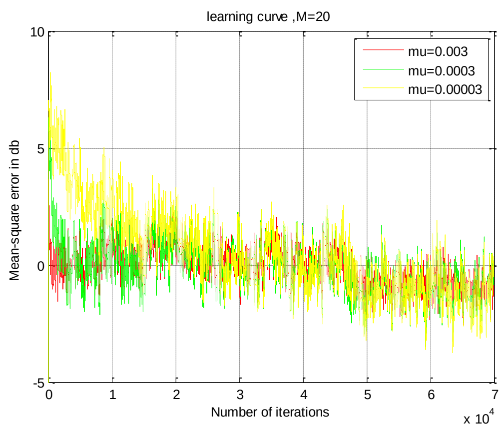

Figure 9. learning curve 4

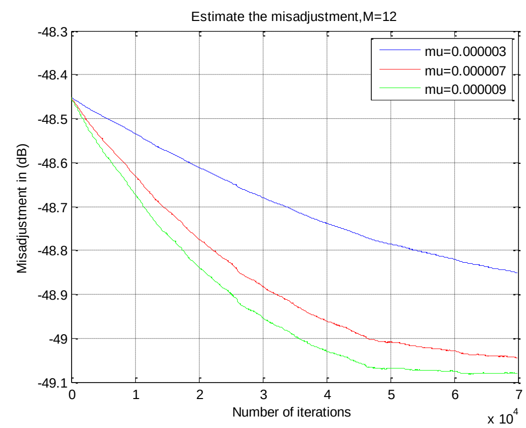

Figure 12. misadjustment curve 3

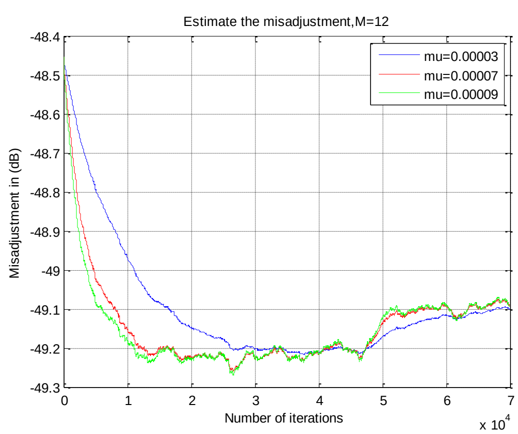

Figure10. misadjustment curve 1

Result and discussion

The graphs above shows the result obtained when we design the adaptive filtering. Surface contour, frequency response, learning curves and misalignment curves all are obtained and shown with best result. The primary measure of performance in the LMS adaptive filter is the mean square error, represented by J(n) = E[|e(n)|2]. For a certain adaptive filter with step size μ, the MSE can be described as a function of the number of iterations. The MSE tends to decrease towards the optimal MSE with increasing number of iterations, but it can never totally converge due to excess MSE caused by the tap weight error the figures give above show the learning curves of least mean square at different values of step size.

A simple simulation was developed to demonstrate the ability of the LMS algorithm to remove unwanted noise from signal. This evaluation shows the number of iterations for noisy tonal signal to mean square error for various step-size parameters. We know that for the large step-size μ=0.1 the filter becomes unstable resulting in the large error output and error signals. For a relatively small μ = 0.0003, the signal has not yet converged after a number of iterations, evidenced by the tapered error signal. The step-size of 0.0003 is stable and closely approximates the desired signal at 1000 samples. The results of the simple simulation give insight into what range of μ values give a reasonable convergence time and stability for further analysis of a more realistic simulation. The result also shows the improvement of signal to noise ratio where it can be seen from the table when mean square error is reduced the stability is increase and vise versa.

Table 1. LMS comparison

S.N	algorithm	MSE	%noise reduction	stability
1	LMS	2.5e-002	91.6%	Highly stable
2	NLMS	2.1e-002	93.8%	high

Conclusion

The implementation of algorithms was successfully achieved, with results that have a really good response as shown in the previous figures. The simulation results show that LMS algorithm give good results in noise cancelling. To complete the task of noise reduction LMS filtering results is relatively good, the requirements length of filter is relatively short, it has a simple structure and small operation and is easy to realize. The signal to noise ratio was measured as given in table 1. However, when the adaptive filter operates in a non-stationary environment, the bottom of the error performance surface continually moves, while the orientation and curvature of the surface may be changing too.

References

[1] J. Benesty, *et. al.* , "Adaptive Filtering Algorithms for Stereophonic Acoustic Echo Cancellation**",** International Conference on Acoustics, Speech, and Signal Processing, 1995, vol.5, Page(s): 3099 – 3102.

[2] Bernard Widrow, *et. al.* "Adaptive Noise Cancelling: Principles and Applications", Proceedings of the IEEE, 1975, Vol.63 , No. 12 , Page(s): 1692 – 1716.

[3] Simon Haykin, "Adaptive Filter Theory", Prentice Hall, 4th edition.

[4] Ying He, *et. al.* "The Applications and Simulation of Adaptive Filter in Noise Canceling", 2008 International Conference on Computer Science and Software Engineering, 2008, Vol.4, Page(s): 1 – 4.

[5] Paulo S.R. Diniz, "Adaptive Filtering: Algorithms and Practical Implemetations" ,ISBN 978-0-387-31274-3, Kluwer Academic Publisher © 2008 Springer Science+Business Media, LLC, pp.77-195.

[6] Slock, D.T.M., "On the convergence behavior of the LMS and the normalized LMS algorithms", IEEE Transactions on Signal Processing, 1993,Vol. 41, Issue 9, pp. 2811-2825.

[7] Abhishek Tandon, M. Omair Ahmad, **"**An efficient, low-complexity, normalized LMS algorithm for echo cancellation" The 2nd Annual IEEE Northeast Workshop on Circuits and Systems, 2004. NEWCAS 2004, Page(s): 161 – 164.

[8] Sanaullah Khan, M.Arif and T.Majeed, "Comparison of LMS, RLS and Notch Based Adaptive Algorithms for Noise Cancellation of a typical Industrial Workroom", 8th International Multitopic Conference, 2004, Page(s): 169 – 173.

[9] Yuu-Seng Lau, Zahir M. Hussian and Richard Harris, "Performance of Adaptive Filtering Algorithms: A Comparative Study", Australian Telecommunications, Networks and Applications Conference (ATNAC), Melbourne, 2003.